# Dual Enrollment Pre-Calculus Packet (Mrs. Thacker)

## April 1st - April 15th

*Please know how much I miss teaching you and seeing all your wonderful faces in class. Please stay safe and I can't wait until we are all back together again ☺

If you need to contact me, here are the methods:

Email: thackerc@santarosa.k12.fl.us

Remind: private message

Microsoft Teams

*I will be available for help and questions Monday-Friday 9:00 AM - 11:00 AM and 2:00 PM - 3:30 PM.

*You will need to use your Pre-Calculus textbook to complete each homework assignment. You can check most of your answers in the back of the book. If you can access Microsoft teams or my Remind, or my website, I will post answers to the homework worked out on each site.

*If you need hard copies and/or anything from the school - please contact Mrs. Parr at 916-4110 to schedule an appointment to retrieve any items from the school.

*In this packet you will have a blank set of notes for each section, along with the filled in notes. Fill the notes out on your own to learn the concept. After you fill in the notes, complete the homework assignment associated with the section.

Miss you,

Mrs. Thacker

### Week 1 - April 2nd - 3rd

| Section | Assignment |
|---|---|
| 2.8 | p.165-167 #1-8 all, 17, 21-24 all |
| 2.2 | p.109-111 #9-16all, 21, 25, 35, 41, 51, 57, 73, 79, 109 |

### Week 2 - April 6th - 10th

| Section | Assignment |
|---|---|
| 2.3a | p.124-125 #9-31 odd, 45, 49 |
| 2.3b | p.125-127 #55, 59, 85, 95, 107 |

### Week 3 - April 13th - 15th

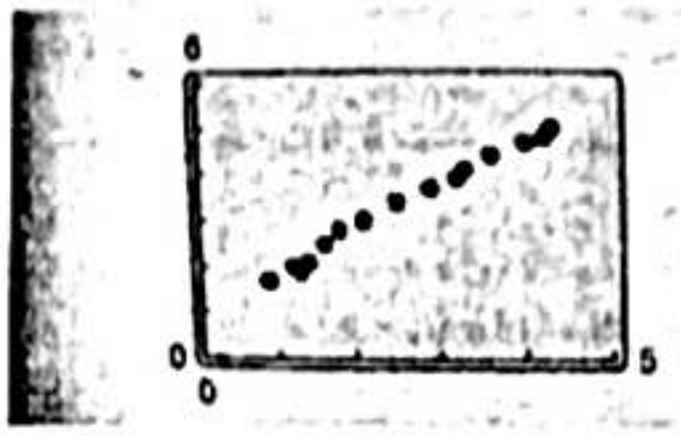
| Section | Assignment |
|---|---|
| 2.4a | p.133 #7-49 odd |
| 2.4b | p.133-134 #51-83 odd |

# 2.8 Quadratic Models

Recall: A scatterplot can be used to show a relationship between two variables.

Decide which model bests fits the data: linear model, quadratic model, or neither.

____________ ____________ ____________



Recall: Linear model - $f(x) = mx + b$

Quadratic model - $f(x) = ax^2 + bx + c$

**Example 1:** The time y (in seconds) required to attain a speed of x miles per hour from a standing start for an automobile is shown in the table.

| Speed, $x$ | 0 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| Time, $y$ | 0 | 1.4 | 2.6 | 3.8 | 4.9 | 6.3 | 8.0 | 9.9 |

(a) Create a scatterplot using your calculator.

(b) Use the regression feature of the graphing utility to find the model that bests fits the data.

(c) Use the model to estimate how long it takes the automobile (from start) to reach a speed of 55 mph.

Example 2: The table shows the numbers y (in thousands) of full-size, alternative fueled vehicles in the use from 2005 - 2011. Use the regression feature of a calculator to find the linear AND quadratic model for the data. Determine which model better fits the data.

DATA

| Year | Number of alternative fueled vehicles (in thousands), $y$ |
|---|---|
| 2005 | 19.2 |
| 2006 | 31.3 |
| 2007 | 44.9 |
| 2008 | 59.8 |
| 2009 | 64.2 |
| 2010 | 72.1 |
| 2011 | 81.3 |

Spreadsheet at LarsonPrecalculus.com

Linear

______________________________

Quadratic

______________________________

**Technology Tip**

When you use the *regression* feature of a graphing utility, the program may output an "$r^2$-value." This $r^2$-value is the coefficient of determination of the data and gives a measure of how well the model fits the data. The coefficient of determination for the linear model in Example 4 is $r^2 \approx 0.9608$, and the coefficient of determination for the quadratic model is $r^2 \approx 0.9962$. Because the coefficient of determination for the quadratic model is closer to 1, the quadratic model better fits the data.

2.8 HW Assignment #1

pages 165-167 #1-8 all, 17, 21-24 all

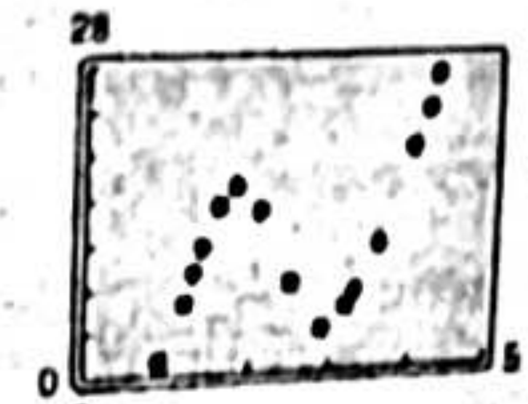
Key

## 2.8 Quadratic Models

Recall: A scatterplot can be used to show a relationship between two variables.

Decide which model bests fits the data:
linear model, quadratic model, or neither.

linear     quadratic     neither



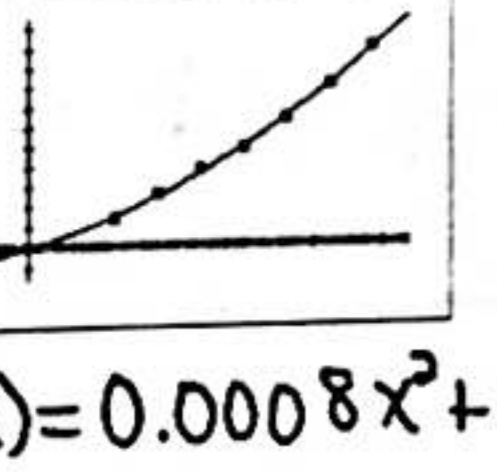
Recall: Linear model - $f(x) = mx + b$

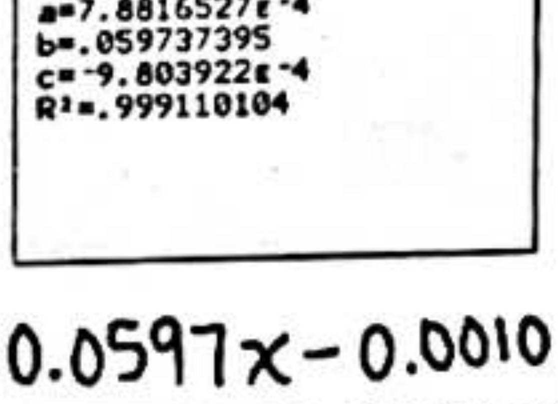
Quadratic model - $f(x) = ax^2 + bx + c$

**Example 1:** The time y (in seconds) required to attain a speed of x miles per hour from a standing start for an automobile is shown in the table.

| DATA | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Speed, x** | 0 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| **Time, y** | 0 | 1.4 | 2.6 | 3.8 | 4.9 | 6.3 | 8.0 | 9.9 |

(a) Create a scatterplot using your calculator.

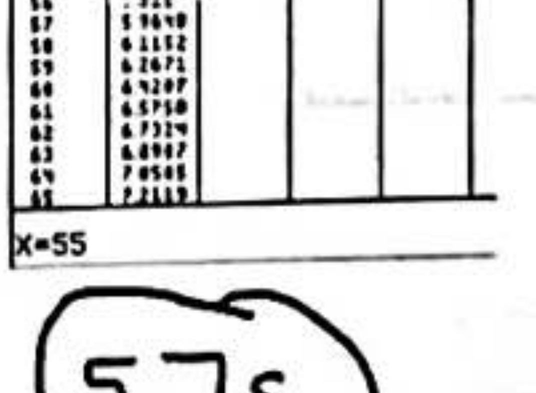


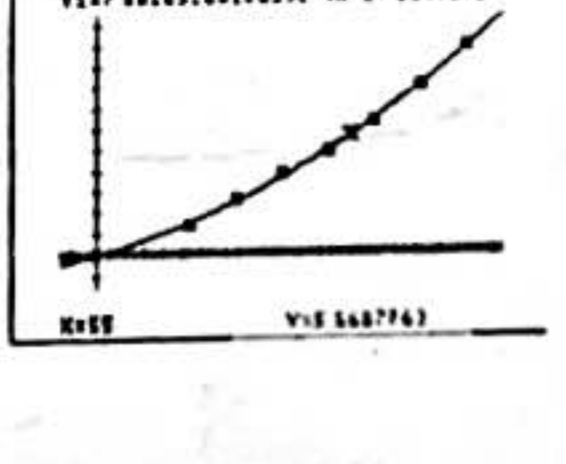


```
NORMAL FLOAT AUTO REAL RADIAN MP
QuadReg
y=ax²+bx+c
a=7.8816527ᴇ-4
b=.059737395
c=-9.803922ᴇ-4
R²=.999110104
```

(b) Use the regression feature of the graphing utility to find the model that bests fits the data.

$f(x)=0.0008x^2+0.0597x-0.0010$





(c) Use the model to estimate how long it takes the automobile (from start) to reach a speed of 55 mph.

X=55

$\boxed{5.7s}$

**Example 2:** The table shows the numbers y (in thousands) of full-size, alternative fueled vehicles in the use from 2005 - 2011. Use the regression feature of a calculator to find the linear AND quadratic model for the data. Determine which model better fits the data.

| Year | Number of alternative fueled vehicles (in thousands), y |
|---|---|
| 2005 | 19.2 |
| 2006 | 31.3 |
| 2007 | 44.9 |
| 2008 | 59.8 |
| 2009 | 64.2 |
| 2010 | 72.1 |
| 2011 | 81.3 |

Quadratic

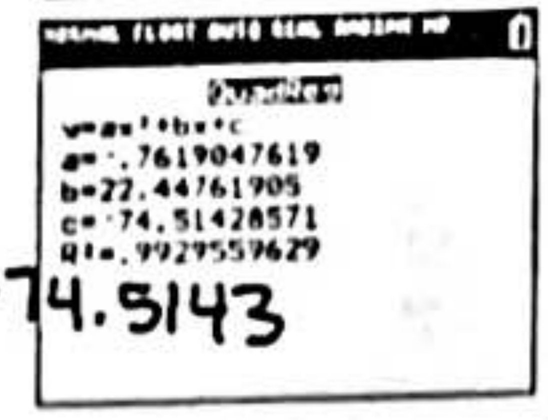
*Linear

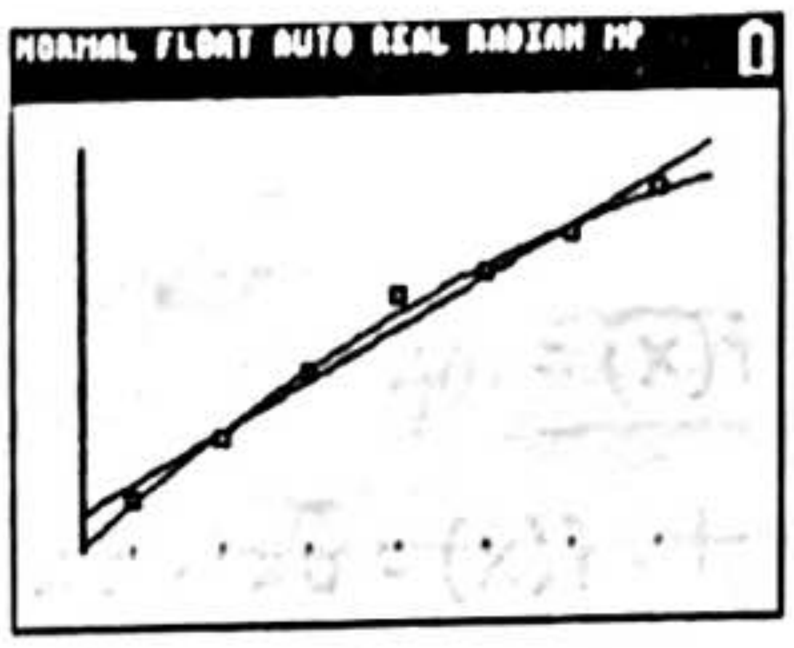
$f(x) = 10.2571x - 28.8$


```
NORMAL FLOAT AUTO REAL RADIAN MP
LinReg
y=ax+b
a=10.25714286
b=⁻28.8
r²=.9767874568
r=.9883255824
```

*Quadratic

$f(x) = -0.7619x^2 + 22.4476x - 74.5143$


```
NORMAL FLOAT AUTO REAL RADIAN MP
QuadReg
y=ax²+bx+c
a=⁻.7619047619
b=22.44761905
c=⁻74.51428571
R²=.9929559629
```



**Technology Tip**

When you use the *regression* feature of a graphing utility, the program may output an "$r^2$-value." This $r^2$-value is the **coefficient of determination** of the data and gives a measure of how well the model fits the data. The coefficient of determination for the linear model in Example 4 is $r^2 \approx 0.9608$, and the coefficient of determination for the quadratic model is $r^2 \approx 0.9962$. Because the coefficient of determination for the quadratic model is closer to 1, the quadratic model better fits the data.

2.8 HW Assignment #1

pages 165-167 #1-8 all, 17, 21-24 all

# 2.2 Polynomial Functions of Higher Degree

The graphs of polynomial functions are _______________.

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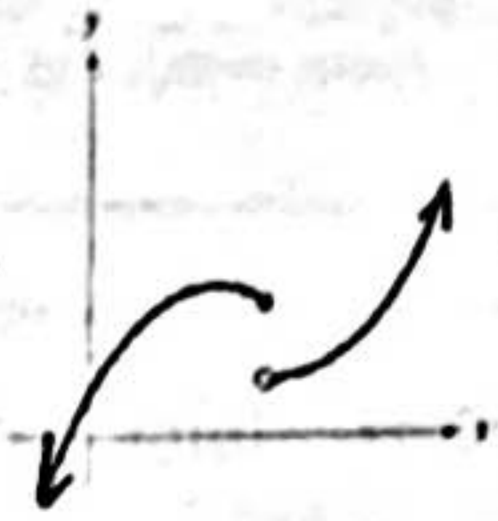
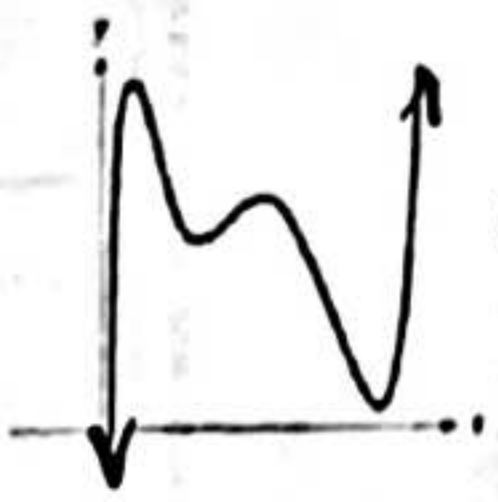
Ex 1: Given the graphs below, which ones represent polynomial functions?

a.

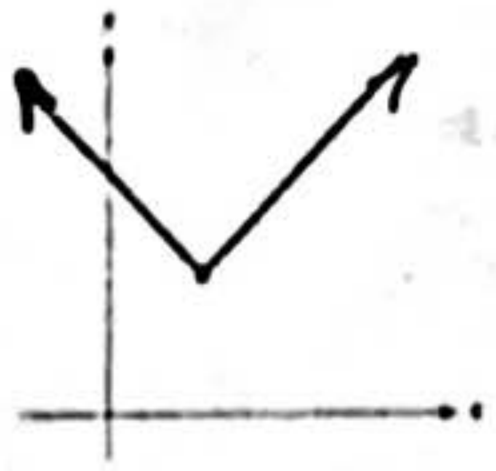
b.



c.



d.



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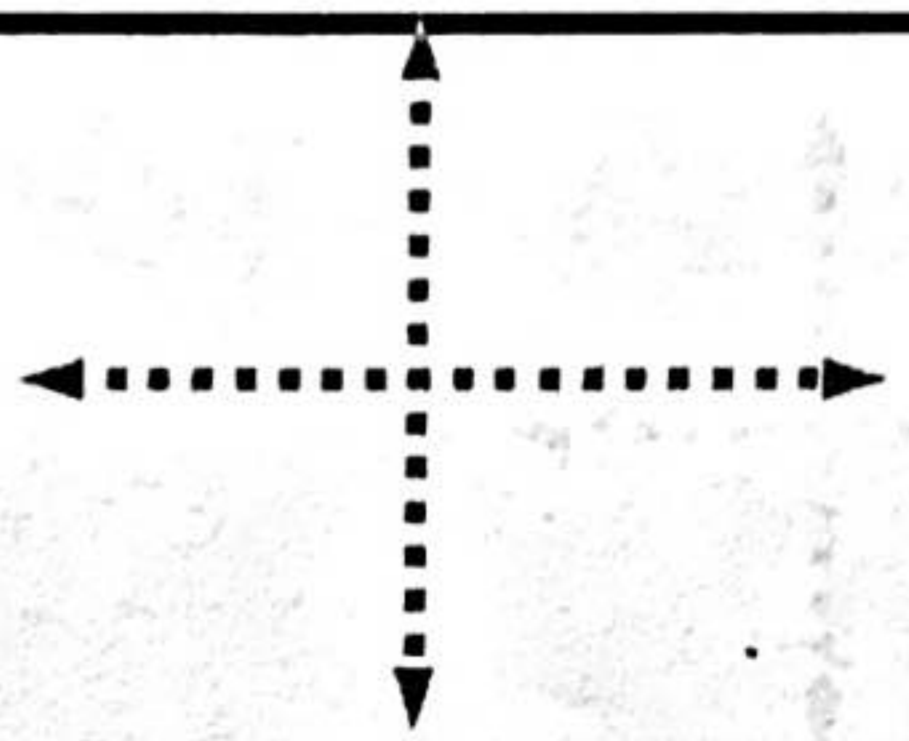
Ex 2: Graph $f(x) = x^3$, find each:

domain: ______ increasing: _______

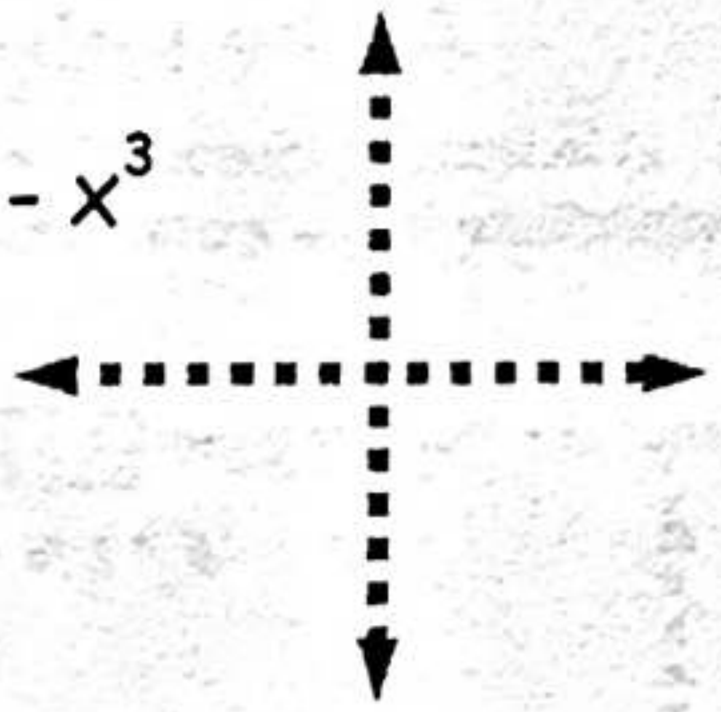
range: _______ even or odd? ______

intercept: _______ symmetry ________

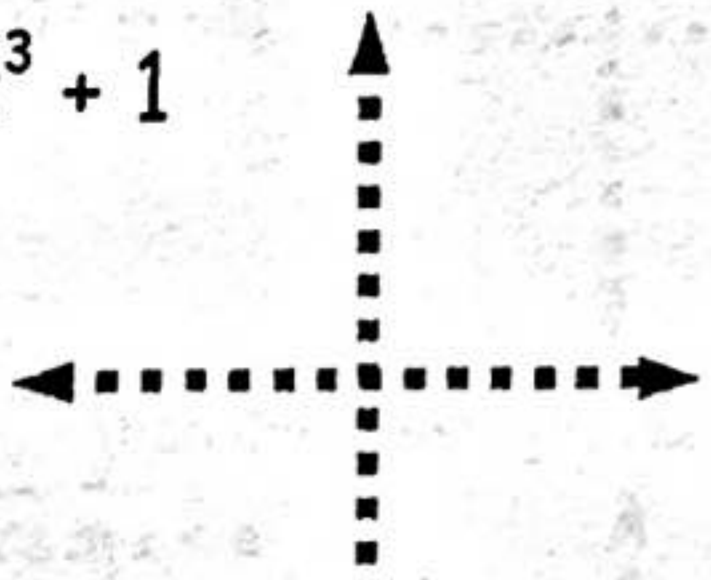


Ex 3: Sketch each:

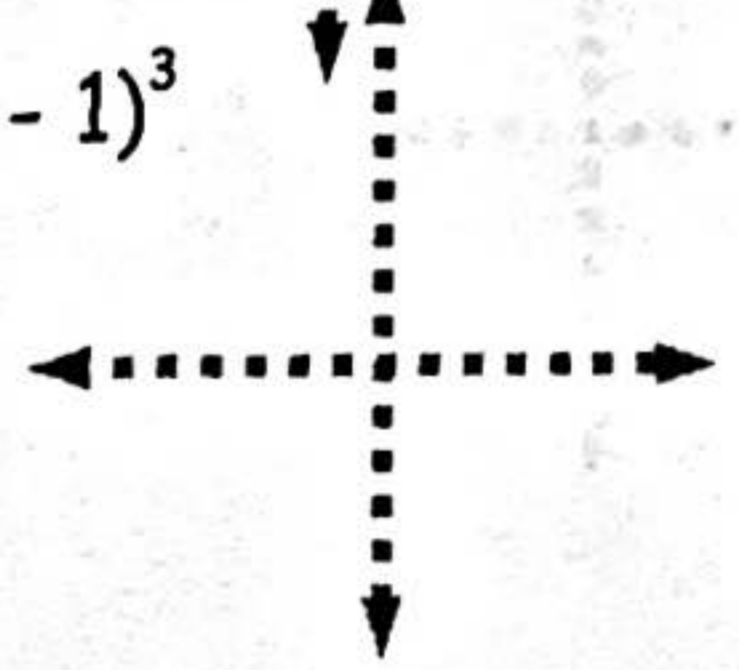
a. $g(x) = -x^3$



b. $h(x) = x^3 + 1$



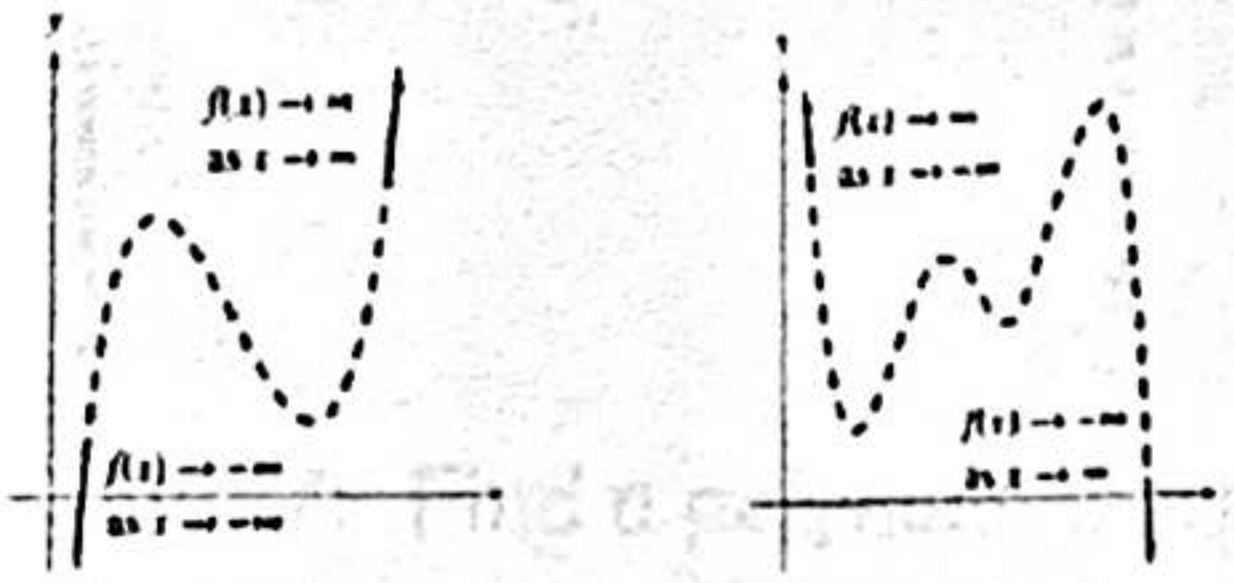
c. $k(x) = (x - 1)^3$



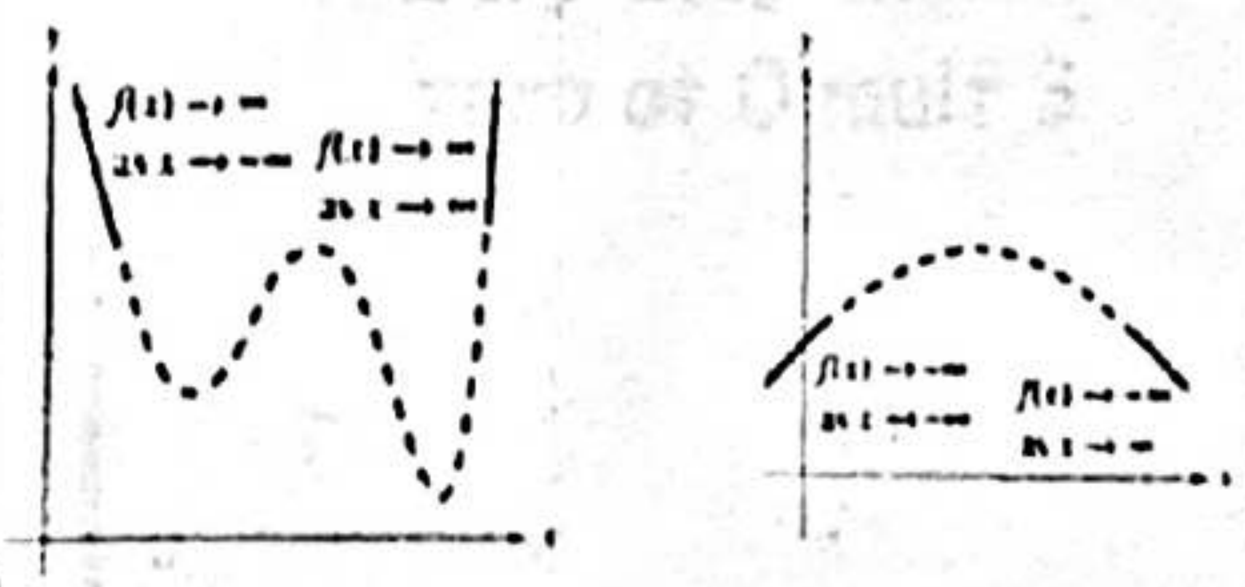
## Leading Coefficient Test

Given: $f(x) = a_nx^n + ... + a_1x + a_0$, where $a_n \neq 0$ then the graph of even and odd functions have the following behaviors:

Odd



Even



## Zeros of a Polynomial Function

Given a polynomial function of degree n, the following are true:

1. The function has at most ____ real zeros.
2. The graph has at most ______ relative extrema (relative min or relative max).

## Real Zeros of a Polynomial Function

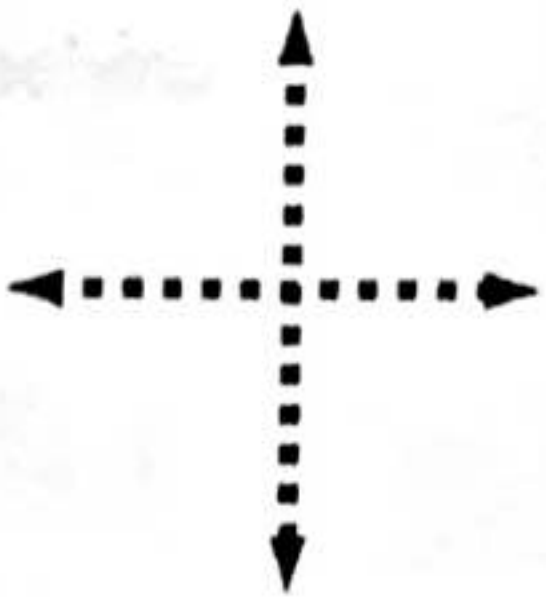
When f is a polynomial function and a is a real number, the following are true:

1. x = a is a ______ of the function.
2. x = a is a ________ when f(x) = 0.
3. (x - a) is a _______ of f(x).
4. (a, 0) is an ____________ of the graph f.

## Ex 4: Find the zeros of $f(x) = x^3 - x^2 - 2x$

Algebraically

Graphically



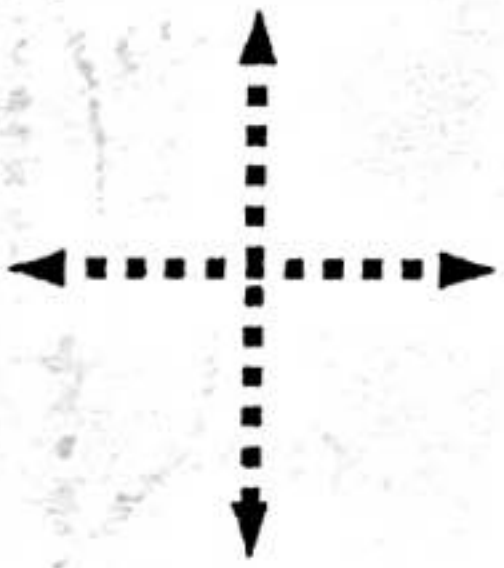
## Ex 5: Analyze: $f(x) = -2x^4 + 2x^2$

## Repeated Zeros

For a polynomial function, a factor of $(x - a)^k$, $k > 1$, yields a repeated zero x = a of multiplicity k.

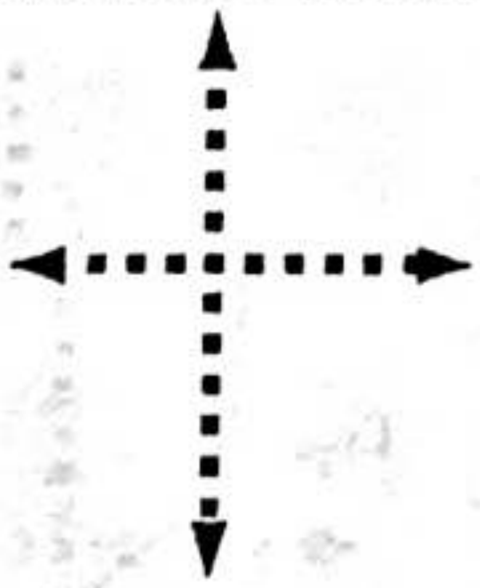
1. If k is odd, then the graph crosses the x-axis at x = a.

Example: $f(x) = (x - 2)^3$

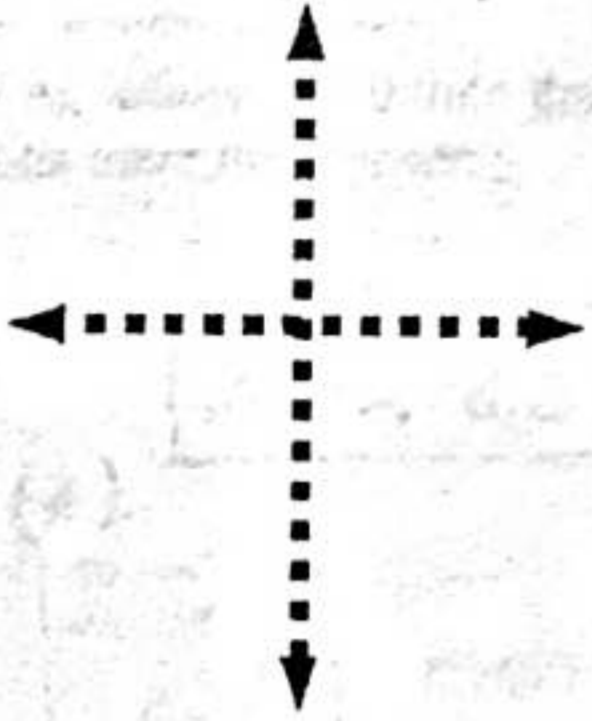


2. If k is even, then the graph touches (bounces) the x-axis at x = a.

Example: $f(x) = (x - 2)^4$



## Ex 6: Graph: $f(x) = x^2(x + 3)^3$



## Ex 7: Find a polynomial function

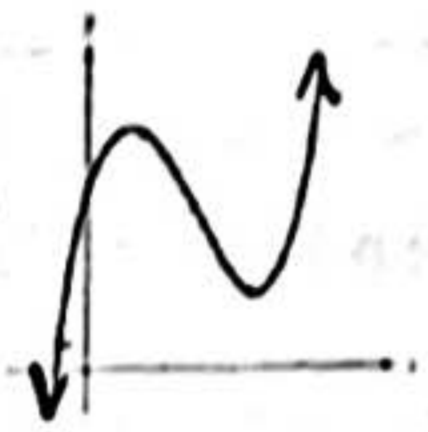
with: zero at $-\frac{1}{2}$ mult 1
zero at 3 mult 2
zero at 0 mult 3

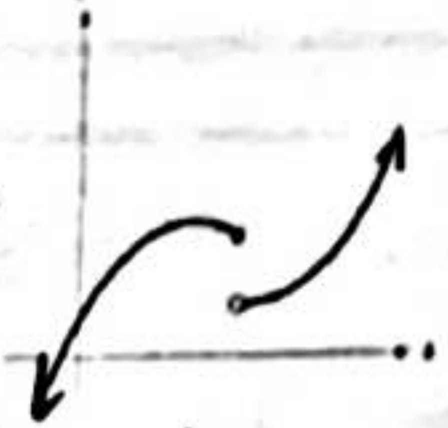
## 2.2 Polynomial Functions of Higher Degree

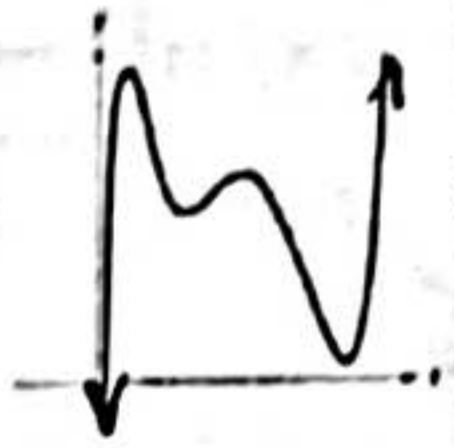
The graphs of polynomial functions are continuous.

---

Ex 1: Given the graphs below, which ones represent polynomial functions?

a.

yes, continuous

b.

No! not continuous, break

c.

Yes, continuous, smooth and rounded

d.

No, no sharp turns

---

Ex 2: Graph $f(x) = x^3$, find each:

domain: $(-\infty, \infty)$

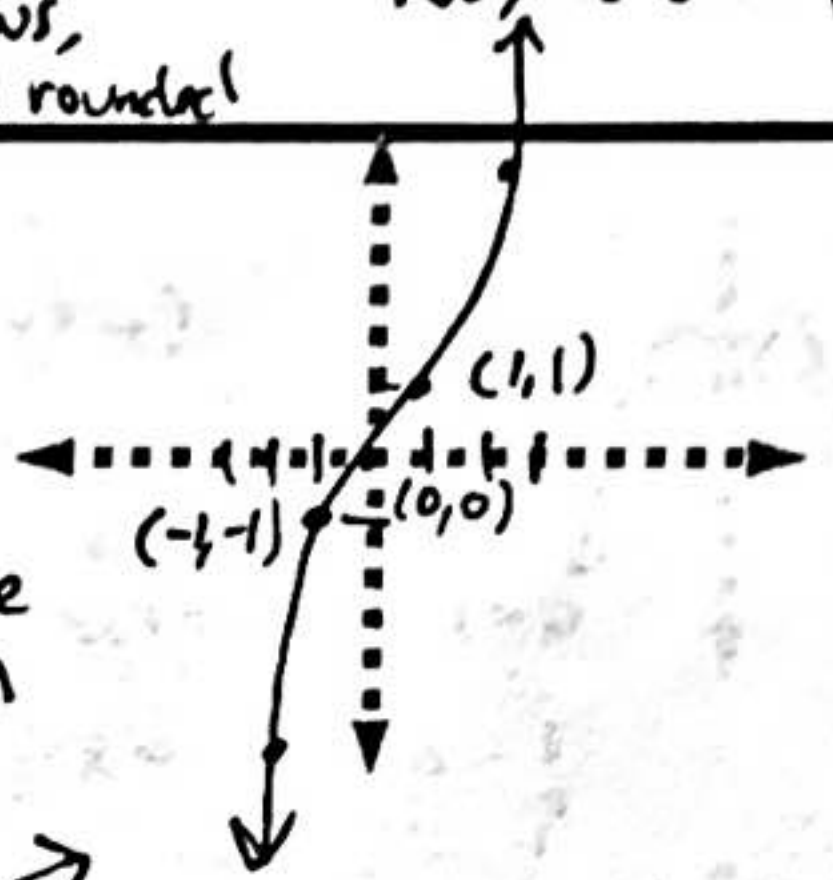
range: $(-\infty, \infty)$

intercept: (0,0)

increasing: $(-\infty, \infty)$

even or odd? odd

symmetry across the origin



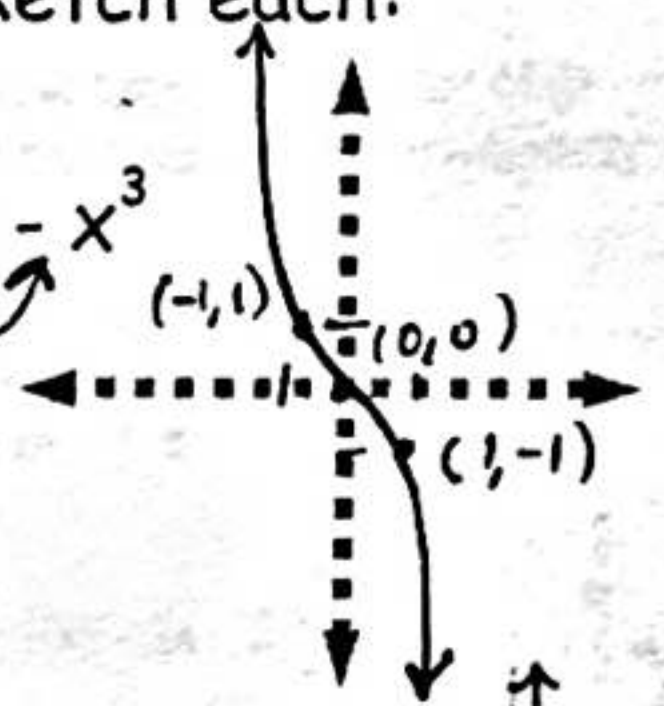
| x | f(x) |
|---|---|
| -3 | -27 |
| -2 | -8 |
| -1 | -1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |

use parent function $f(x) = x^3$

Ex 3: Sketch each:

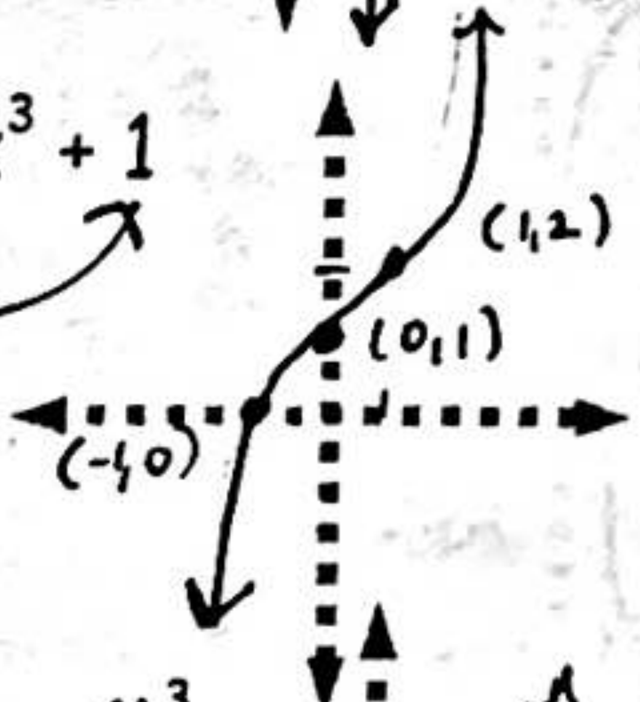
a. $g(x) = -x^3$

reflect across x-axis



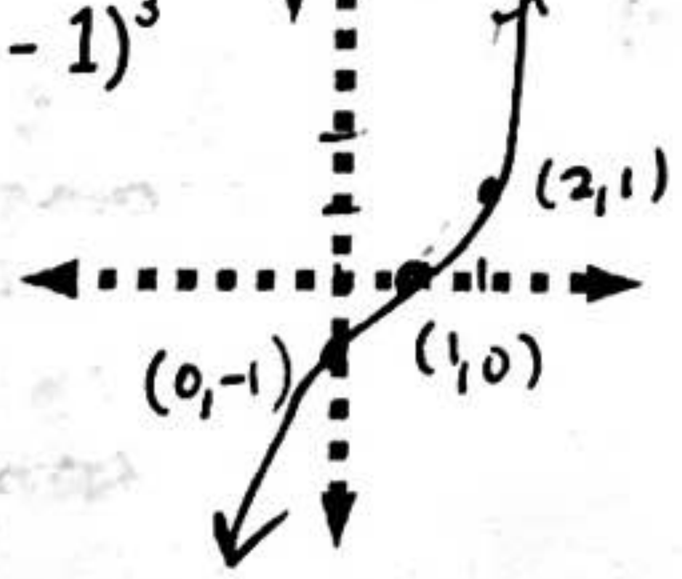
b. $h(x) = x^3 + 1$

vertical shift 1 unit up



c. $k(x) = (x - 1)^3$

horizontal shift one unit to the right

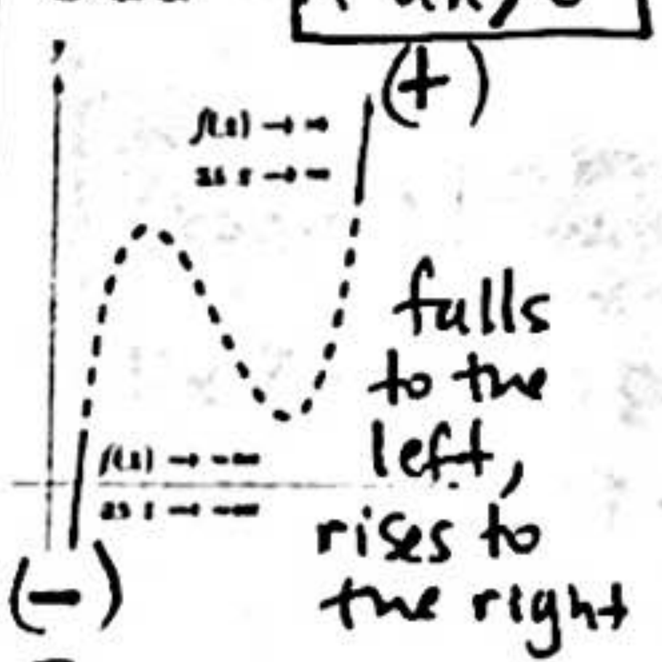


### Leading Coefficient Test

Given: $f(x) = a_nx^n + \ldots + a_1x + a_0$, where $a_n \neq 0$ then the graph of even and odd functions have the following behaviors:

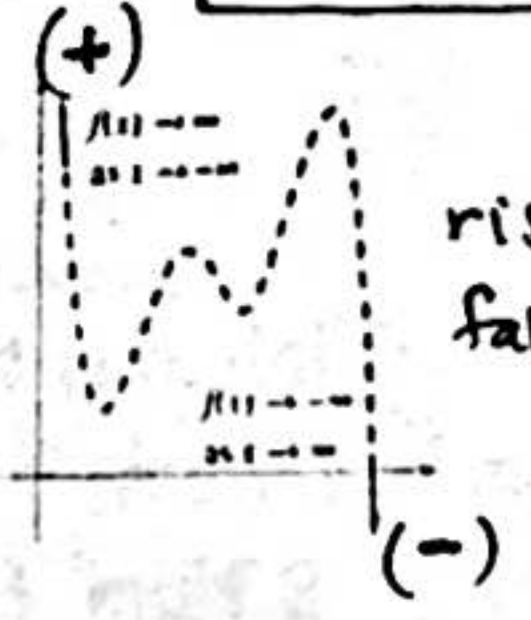
Odd

if $a_n > 0$



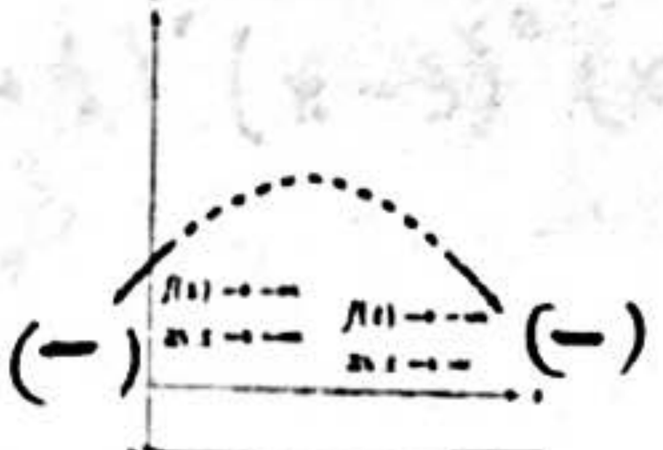
falls to the left, rises to the right

if $a_n < 0$



rises to the left, falls to the right

Even



if $a_n > 0$

graph rises on both left and right

if $a_n < 0$

graph falls on both left and right

## Zeros of a Polynomial Function

Given a polynomial function of degree n, the following are true:

1. The function has at most n real zeros.
2. The graph has at most (n-1) relative extrema (relative min or relative max).

## Real Zeros of a Polynomial Function

When f is a polynomial function and a is a real number, the following are true:

1. x = a is a zero of the function.
2. x = a is a solution when f(x) = 0.
3. (x - a) is a factor of f(x).
4. (a, 0) is an x-intercept of the graph f.

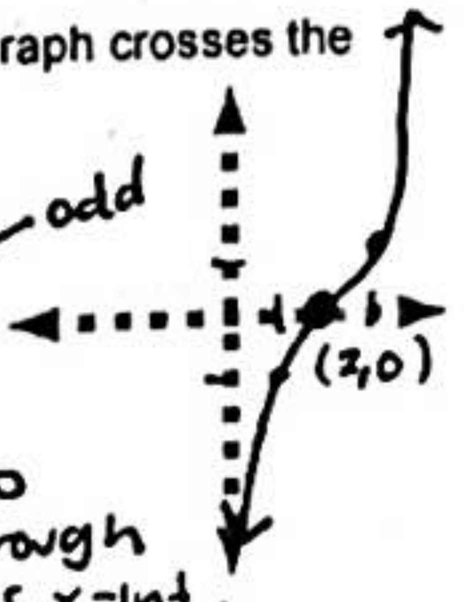
## Repeated Zeros

For a polynomial function, a factor of $(x - a)^k$, $k > 1$, yields a repeated zero x = a of multiplicity k.

1. If k is odd, then the graph crosses the x-axis at x = a.

odd

Example: $f(x) = (x - 2)^3$

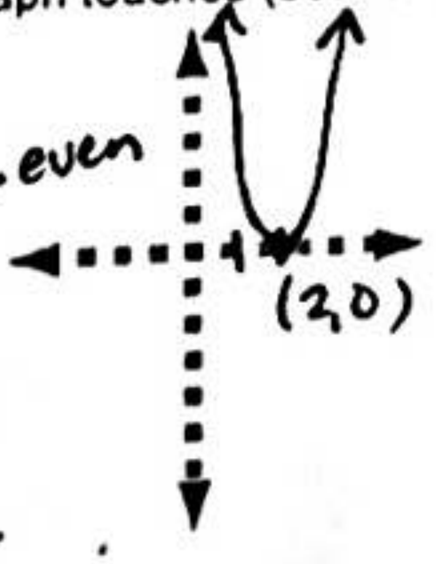


x = 2
is a zero
graph passes through
this x-int.

2. If k is even, then the graph touches (bounces) the x-axis at x = a.

even

Example: $f(x) = (x - 2)^4$



x = 2
is a zero
graph touches or
bounces at this
x-intercept

## Ex 4: Find the zeros of $f(x) = x^3 - x^2 - 2x$

Algebraically

set f(x) = 0

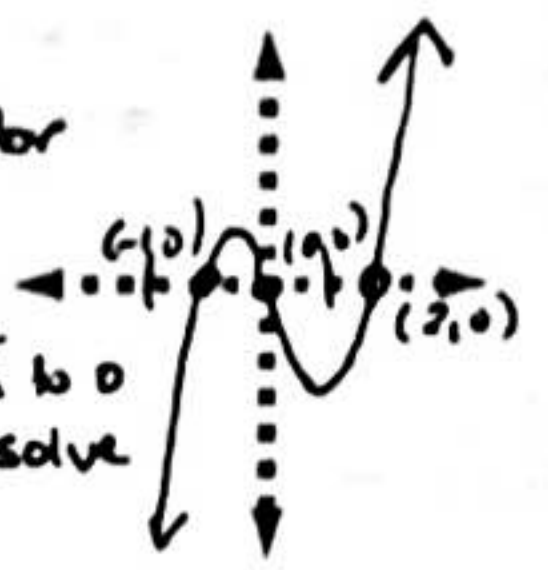
$0 = x^3 - x^2 - 2x$ factor

$0 = x(x^2 - x - 2)$

$0 = x(x-2)(x+1)$ set each to 0 then solve

zeros x = 0
x = 2
x = -1

Graphically

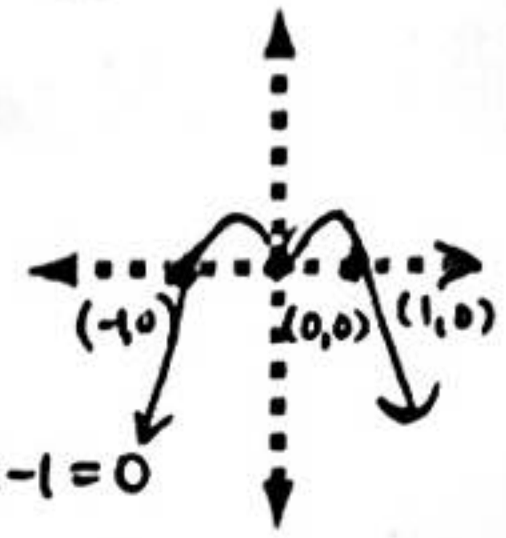


## Ex 5: Analyze: $f(x) = -2x^4 + 2x^2$

$0 = -2x^4 + 2x^2$

$0 = -2x^2(x^2 - 1)$

$0 = -2x^2(x+1)(x-1)$

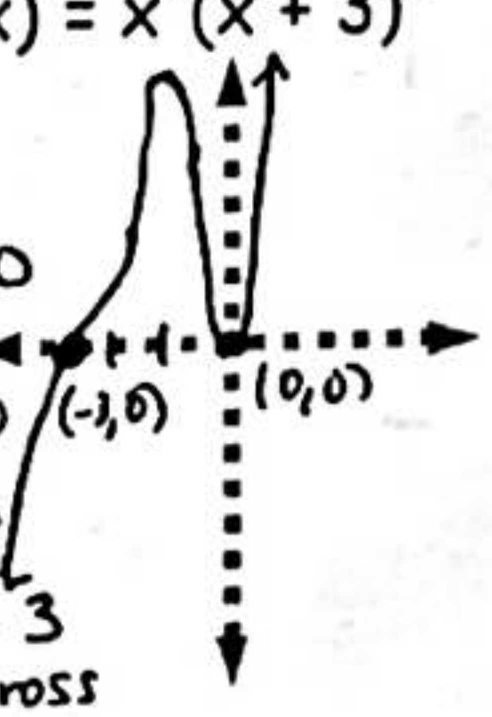


| | | |
|---|---|---|
| $-2x^2 = 0$ | $x+1=0$ | $x-1=0$ |
| $x^2 = 0$ | $x = -1$ | $x = 1$ |
| $x = 0$ | mult 1<br>pass through | mult 1<br>pass through |
| mult 2<br>will bounce | | |

## Ex 6: Graph: $f(x) = x^2(x + 3)^3$

$0 = x^2(x+3)^3$

| | |
|---|---|
| $x^2 = 0$ | $(x+3)^3 = 0$ |
| $x = 0$ | $x+3 = 0$ |
| | $x = -3$ |
| mult 2<br>(bounce) | mult 3<br>will cross<br>x-axis |



## Ex 7: Find a polynomial function

with: zero at $-\frac{1}{2}$ mult 1
zero at 3 mult 2
zero at 0 mult 3

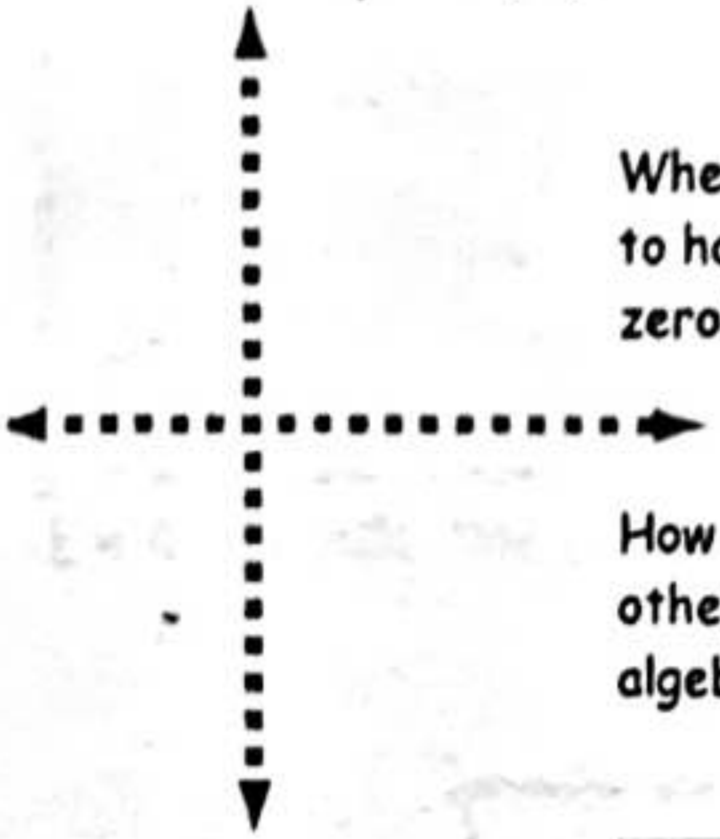
final poly has leading power of 6

$$f(x) = (x + \tfrac{1}{2})^1 (x-3)^2 (x-0)^3$$
$$= (x+\tfrac{1}{2})(x-3)(x-3)x^3$$
$$= (x+\tfrac{1}{2})(x^2-6x+9)x^3$$
$$= (x^3 - 6x^2 + 9x + \tfrac{1}{2}x^2 - 3x + \tfrac{9}{2})x^3$$
$$= x^3(x^3 - \tfrac{11}{2}x^2 + 6x + \tfrac{9}{2})$$

$$\boxed{f(x) = x^6 - \tfrac{11}{2}x^5 + 6x^4 + \tfrac{9}{2}x^2}$$

# 2.3a Real Zeros of Polynomial Functions

Ex 1: Graph $f(x) = 6x^3 - 19x^2 + 16x - 4$



**Where does it appear to have a whole number zero?** ________

**How can we find the other two zero values algebraically?**

________________

**Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, then use the result to factor the polynomial completely.**

$f(x) = (\qquad)^2\, q(x)$

Let's use long division to find q(x)!

Ex 2: Divide the polynomial $3x^2 + 19x + 28$ by $x + 4$, then factor the polynomial completely.

Ex 3: Divide $8x^3 - 1$ by $2x - 1$

**The Division Algorithm**

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}}\underbrace{q(x)}_{\text{Quotient}} + \underbrace{r(x)}_{\text{Remainder}}$$

where $r(x) = 0$ *or* the degree of $r(x)$ is less than the degree of $d(x)$. If the remainder $r(x)$ is zero, then $d(x)$ *divides evenly* into $f(x)$.

The Division Algorithm can also be written as

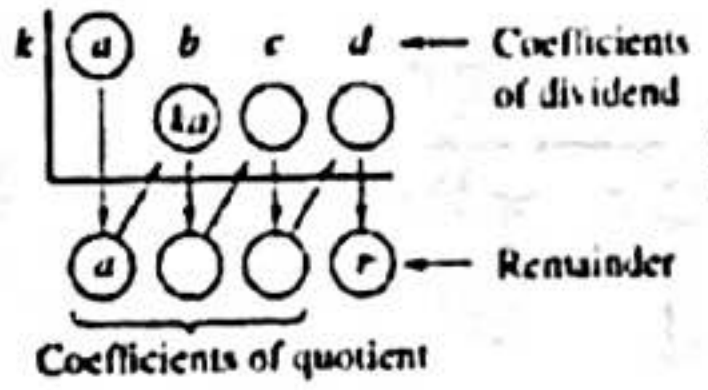
$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

Ex 4: Divide the polynomial $-2 + 3x - 5x^2 + 4x^3 + 2x^4$ by $x^2 + 2x - 3$

Now for the shortcut! Synthetic Division

**Synthetic Division (of a Cubic Polynomial)**

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern.



Vertical pattern: Add terms
Diagonal pattern: Multiply by k

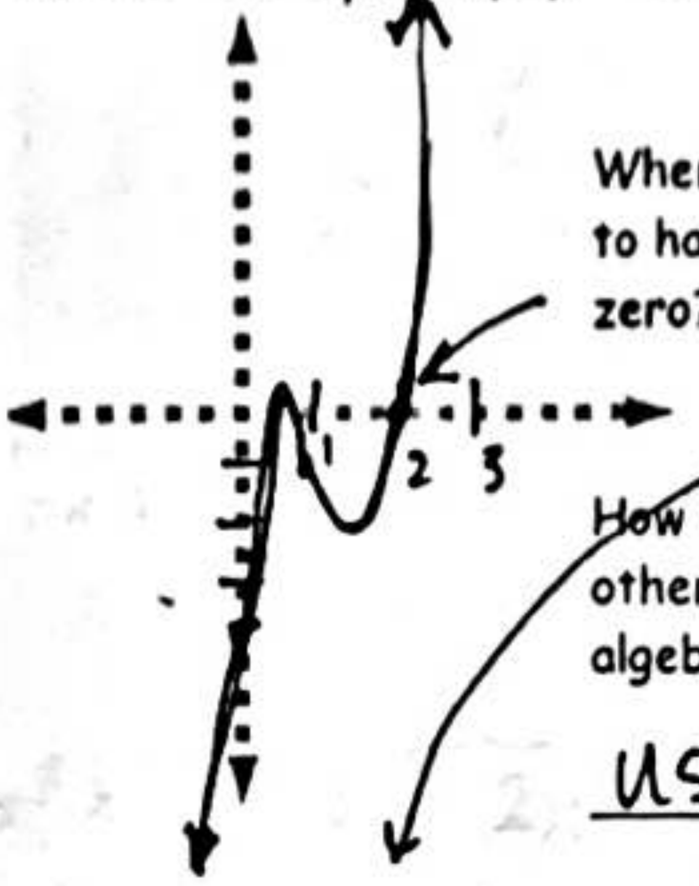
Synthetic Division only works for divisors of the form x - k also x - (- k)

Ex 5: Use synthetic division to divide:

a. $x^4 - 10x^2 - 2x + 4$ by $x + 3$

# 2.3a Real Zeros of Polynomial Functions

Ex 1: Graph $f(x) = 6x^3 - 19x^2 + 16x - 4$



Where does it appear to have a whole number zero? $x = 2$ or $(2,0)$

How can we find the other two zero values algebraically?

use long division

$f(x) = (x-2)\cdot q(x)$

Let's use long division to find q(x)!

Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, then use the result to factor the polynomial completely.

$$\begin{array}{rl}
 & 6x^2 - 7x + 2 \\ \cline{2-2}
x-2 \,\big) & 6x^3 - 19x^2 + 16x - 4 \\
 & -6x^3 \pm 12x^2 \downarrow \\ \cline{2-2}
 & -7x^2 + 16x \\
 & \pm 7x^2 \mp 14x \downarrow \\ \cline{2-2}
 & 2x - 4 \\
 & -2x \pm 4 \\ \cline{2-2}
 & 0
\end{array}$$

now factor $6x^2 - 7x + 2$
$(2x-1)(3x-2)$

Final factored polynomial $f(x) = (x-2)(2x-1)(3x-2$
x-intercepts at $2, \frac{1}{2}, \frac{2}{3}$

Ex 2: Divide the polynomial $3x^2 + 19x + 28$ by $(x + 4)$, then factor the polynomial completely.

$$\begin{array}{rl}
 & 3x + 7 \\ \cline{2-2}
x+4 \,\big) & 3x^2 + 19x + 28 \\
 & -3x^2 \mp 12x \downarrow \\ \cline{2-2}
 & 7x + 28 \\
 & -7x \mp 28 \\ \cline{2-2}
 & 0
\end{array}$$

factored polynomial $(3x+7)(x+4)$

Ex 3: Divide $8x^3 - 1$ by $2x - 1$

$$\begin{array}{rl}
 & 4x^2 + 2x + 1 \\ \cline{2-2}
2x-1 \,\big) & 8x^3 + 0x^2 + 0x - 1 \\
 & -8x^3 \pm 4x^2 \downarrow \\ \cline{2-2}
 & 4x^2 + 0x \\
 & -4x^2 \pm 2x \downarrow \\ \cline{2-2}
 & 2x - 1 \\
 & -2x \pm 1 \\ \cline{2-2}
 & 0
\end{array}$$

☆ When the remainder is zero, that means the polynomial can be factored

**The Division Algorithm**

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}}\underbrace{q(x)}_{\text{Quotient}} + \underbrace{r(x)}_{\text{Remainder}}$$

where $r(x) = 0$ *or* the degree of $r(x)$ is less than the degree of $d(x)$. If the remainder $r(x)$ is zero, then $d(x)$ *divides evenly* into $f(x)$.

**The Division Algorithm can also be written as**

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

Ex 4: Divide the polynomial $\underbrace{-2 + 3x - 5x^2 + 4x^3 + 2x^4}_{\text{write in descending powers}}$ by $x^2 + 2x - 3$

$$\begin{array}{r}
2x^2 \qquad\qquad + 1 \\
x^2+2x-3\;\overline{)\;2x^4 + 4x^3 - 5x^2 + 3x - 2} \\
\underline{-\;2x^4 + {}^{-}4x^3 \pm 6x^2\qquad\quad} \\
x^2 + 3x - 2 \\
\underline{-\;x^2 + {}^{-}2x \pm 3} \\
\boxed{x + 1}
\end{array}$$

remainder $r(x)$

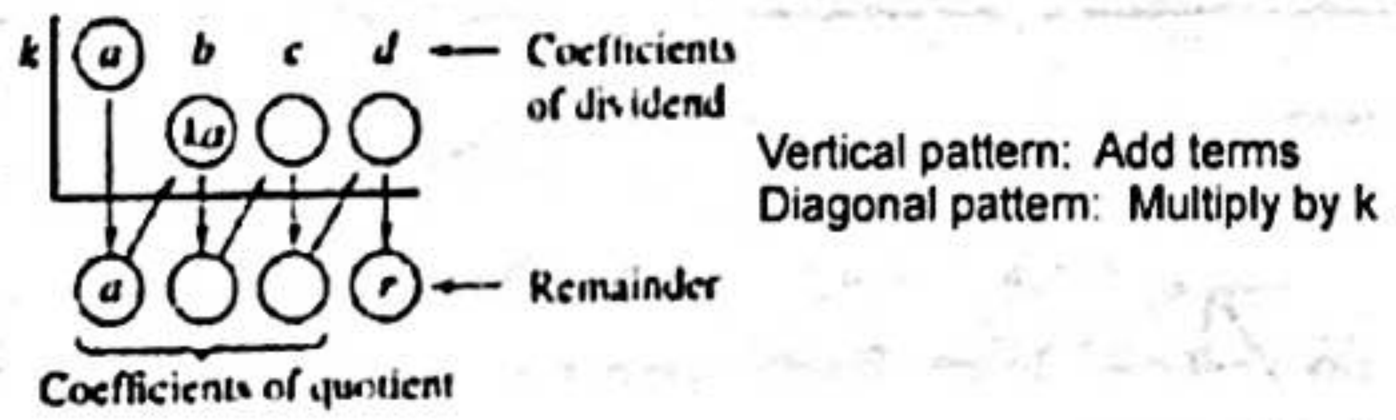
Final answer

$$2x^2+1 + \frac{x+1}{x^2+2x-3}$$

Now for the shortcut! Synthetic Division

**Synthetic Division (of a Cubic Polynomial)**

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern.



Vertical pattern: Add terms
Diagonal pattern: Multiply by k

Synthetic Division only works for divisors of the form x - k also x - (- k)

Ex 5: Use synthetic division to divide:

a. $x^4 \overset{\nearrow}{-}\, 10x^2 - 2x + 4$ by $\underbrace{x + 3}$

$0x^3$

$$\begin{array}{r|rrrrr}
-3 & 1 & 0 & -10 & -2 & 4 \\
 & \downarrow & -3 & 9 & 3 & -3 \\
\hline
 & 1 & -3 & -1 & 1 & \boxed{1}
\end{array}$$

$\underbrace{1 \quad -3 \quad -1 \quad 1}_{\text{Quotient}}$ remainder

$$1x^3 - 3x^2 - 1x + 1 + \frac{1}{x+3}$$

or

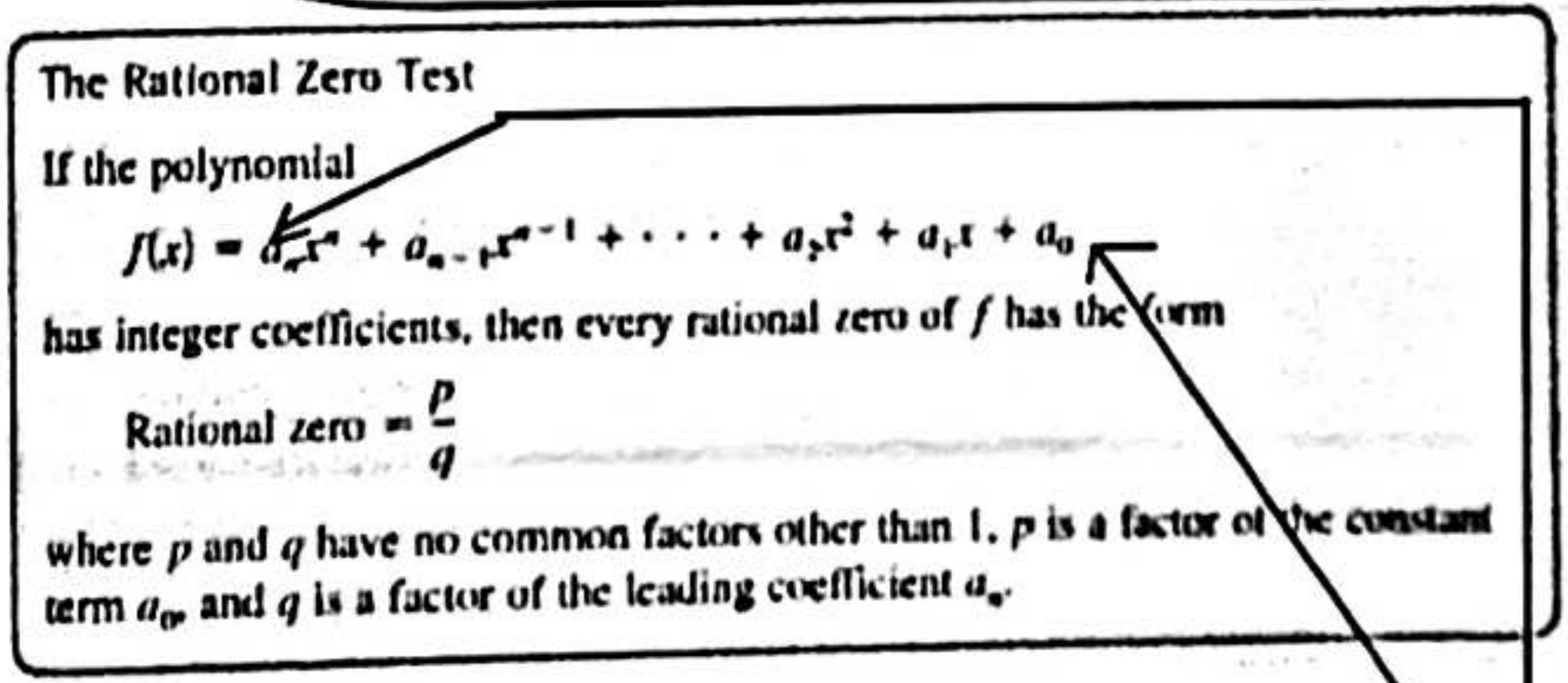
$$\boxed{x^3 - 3x^2 - x + 1 + \frac{1}{x+3}}$$

## 2.3b More on Real Zeros of Polynomial Functions

Ex 1: Given the function: $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$
with factors: $(x + 2), (x - 4)$

| | |
|---|---|
| a. Verify the factors | b. Find the remaining factors |
| c. Write the complete factorization | d. List all zeros |

Rational Zero Test will relate the possible rational zeros of a polynomial having integer coefficients to the leading coefficients and to the constant term of the polynomial.



**The Rational Zero Test**

If the polynomial

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

has integer coefficients, then every rational zero of $f$ has the form

$$\text{Rational zero} = \frac{p}{q}$$

where $p$ and $q$ have no common factors other than 1, $p$ is a factor of the constant term $a_0$, and $q$ is a factor of the leading coefficient $a_n$.

Hint --> Find all factors of the constant term (p),
then all factors of the leading coefficient(q),
Write as $\frac{p}{q}$ and start testing!

Ex 2: Use the Rational Zeros Test to list all possible rational zeros of each function:

a. $f(x) = x^3 - 4x^2 - 4x + 16$

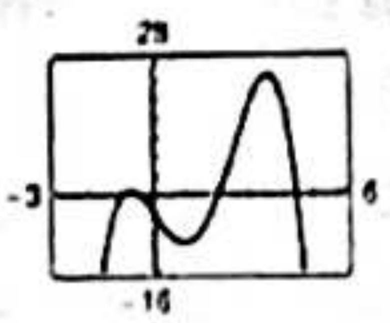
b. $g(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

Ex 3: Find all zeros of $f(x) = 6x^3 - 4x^2 + 3x - 2$

Ex 4: Find all real zeros of $f(x) = 10x^4 - 15x^3 - 16x^2 + 12x$

Ex 5: Given the graph of $y = f(x)$ below, find the zeros. Use a graphing calculator.

$y = -x^4 + 5x^3 - 10x - 4$



# 2.3b More on Real Zeros of Polynomial Functions

Ex 1: Given the function: $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$
with factors: $(x + 2)$, $(x - 4)$

a. Verify the factors

use synthetic ÷

$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

b. Find the remaining factors

$8x^2 + 2x - 3$

$(4x + 3)(2x - 1)$

c. Write the complete factorization

$f(x) = (x+2)(x-4)(4x+3)(2x-1)$

d. List all zeros

$x + 2 = 0$ $x - 4 = 0$ $4x + 3 = 0$ $2x - 1 = 0$

$\left\{-2, 4, -\frac{3}{4}, \frac{1}{2}\right\}$

Rational Zero Test will relate the possible rational zeros of a polynomial having integer coefficients to the leading coefficients and to the constant term of the polynomial.

**The Rational Zero Test**

If the polynomial

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_2x^2 + a_1x + a_0$$

has integer coefficients, then every rational zero of $f$ has the form

$$\text{Rational zero} = \frac{p}{q}$$

where $p$ and $q$ have no common factors other than 1, $p$ is a factor of the constant term $a_0$, and $q$ is a factor of the leading coefficient $a_n$.

Hint --> Find all factors of the constant term (p), then all factors of the leading coefficient(q), Write as $\frac{p}{q}$ and start testing!

Ex 2: Use the Rational Zeros Test to list all possible rational zeros of each function:

a. $f(x) = \overset{q}{1}x^3 - 4x^2 - 4x + \overset{p}{16}$

factors of $p = \pm 1, \pm 2, \pm 4, \pm 16$

factors of $q = \pm 1$

$\frac{p}{q} = \boxed{\pm 1, \pm 2, \pm 4, \pm 16}$

b. $g(x) = \overset{q}{4}x^5 - 8x^4 - 5x^3 + 10x^2 + x - \overset{p}{2}$

factors of $p = \pm 1, \pm 2$

factors of $q = \pm 1, \pm 2, \pm 4$

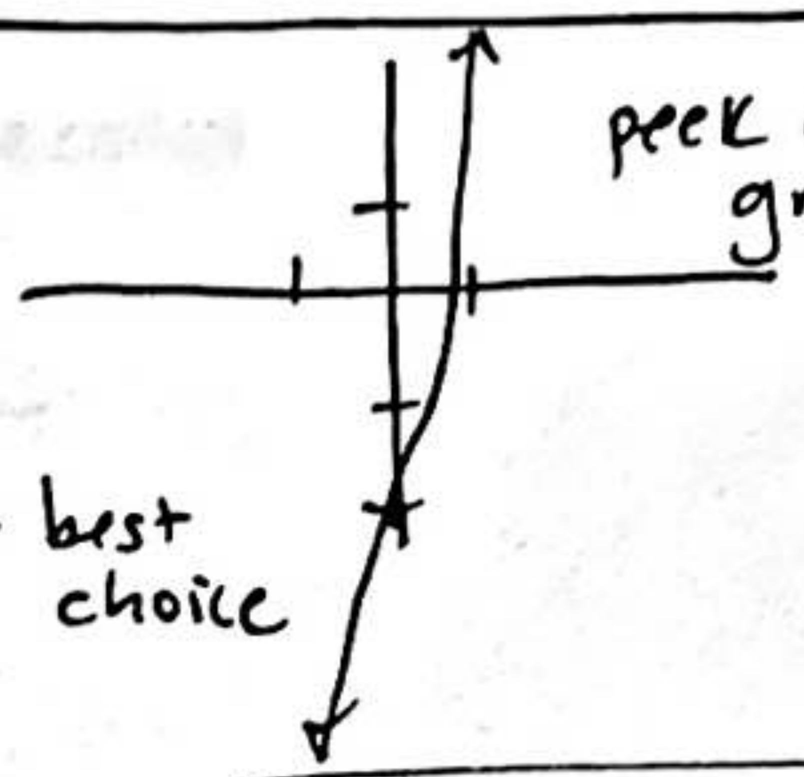
$\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{4}$ so

$\frac{p}{q} = \boxed{\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2}$

Ex 3: Find all zeros of $f(x) = \underset{q}{6}x^3 - 4x^2 + 3x - \underset{p}{2}$

$\frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

$= \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{2}{3}, \pm 2$



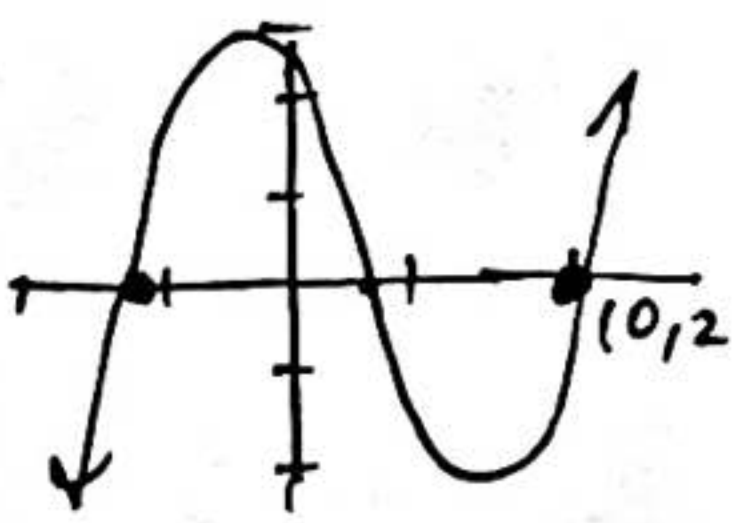
Look at graph, 1 zero, test the choices between 0 and 1, so test $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ and $\frac{2}{3}$ ← best choice

$$\begin{array}{r|rrrr} \frac{2}{3} & 6 & -4 & 3 & -2 \\ & & 4 & 0 & 2 \\ \hline & 6 & 0 & 3 & 0 \end{array} \rightarrow 6x^2 + 0x + 3$$

$\boxed{f(x) = (x - \frac{2}{3})(6x^2 + 3}$

Ex 4: Find all real zeros of $f(x) = 10x^4 - 15x^3 - 16x^2 + 12x$

$f(x) = \underset{\text{Zero is a factor}}{x}(\underset{q}{10}x^3 - 15x^2 - 16x + \underset{p}{12})$



$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 5, \pm 10}$

Look at graph and use 2 as a factor

$$\begin{array}{r|rrrr} 2 & 10 & -15 & -16 & 12 \\ & & 20 & 10 & -12 \\ \hline & 10 & 5 & -6 & 0 \end{array}$$

$0 = 10x^2 + 5x - 6$ → Use quad formula to get $x = \frac{-5 \pm \sqrt{265}}{20}$

All zeros:

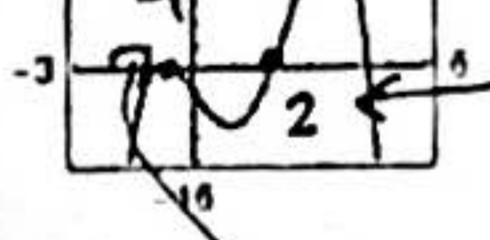
$\{0, 2, \frac{-5 \pm \sqrt{265}}{20}\}$

$\{0, 2, 0.56, -1.06\}$

Ex 5: Given the graph of $y = f(x)$ below, find the zeros. Use a graphing calculator.

$y = -x^4 + 5x^3$
$- 10x - 4$

$y = -x^4 + 5x^3 - 10x - 4$



Test
$$\begin{array}{r|rrrrr} 2 & -1 & 5 & 0 & -10 & -4 \\ & & -2 & 6 & 12 & 4 \\ \hline & -1 & 3 & 6 & 2 & 0 ✓ \end{array}$$

Test
$$\begin{array}{r|rrrr} -1 & -1 & 3 & 6 & 2 \\ & & 1 & -4 & -2 \\ \hline & -1 & 4 & 2 & 0 \end{array}$$

→ $0 = -x^2 + 4x + 2$

Use Quad Form

$\{-1, 2, 2 \pm \sqrt{6}\}$

# Pre-Calc 2.4a Complex Numbers

Recall: $i = \sqrt{-1}$

The imaginary unit "i"

## Definition of a Complex Number

If a and b are real numbers, then the number a + bi is a complex number, and it is said to be written in standard form.

a - real part

b - imaginary part

If b = 0, then the # a + bi = a is a real #.

If b ≠ 0, then the # a + bi is called an imaginary #.

A number of form "bi" - pure imaginary number.

## Equality of Complex Numbers

Two complex numbers a + bi and c + di, written in standard form, are equal to each other if and only if a = c and b = d.

## Operations with Complex Numbers

Two complex numbers a + bi and c + di, written in standard form:

Sum: (a + bi) + (c + di) = (a + c) + (b + d)i

Difference: (a + bi) - (c + di) = (a - c) + (b - d)i

*Properties that apply to complex numbers:

Associative Properties of Addition and Multiplication

Commutative Properties of Addition and Multiplication

Distributive Property of Multiplication over Addition

**Complex Conjugates**
**a + bi and a - bi**

Notice: $(a+bi)(a-bi) = a^2 - abi + abi - b^2i^2$

$= a^2 - b^2(-1)$

$= a^2 + b^2$

## Examples:

1) Find the real numbers a and b such that equation is true.

a + bi = 12 + 5i

2) Write the complex number in standard form.

$2i^2 - 6i$

3) Perform the operation and write result in standard form.

(a) $\left(\frac{3}{4}+\frac{7}{5}i\right)-\left(\frac{5}{6}-\frac{1}{6}i\right)$

(b) $-6(5-3i)$

(c) $(6-2i)(2-3i)$

(d) $(5-4i)^2$

4) Write the complex conjugate - then multiply the numbers.

$-4-\sqrt{3}i$

HW 2.4a:
page 133
#7-49 odd

Key

# Pre-Calc 2.4a Complex Numbers

Recall: $i = \sqrt{-1}$

The imaginary unit "i"

## Definition of a Complex Number

If a and b are real numbers, then the number a + bi is a complex number, and it is said to be written in standard form.

If b = 0, then the # a + bi = a is a real #.

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A number of form "bi" - pure imaginary number.

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Two complex numbers a + bi and c + di, written in standard form, are equal to each other if and only if a = c and b = d.

## Operations with Complex Numbers

Two complex numbers a + bi and c + di, written in standard form:

Sum: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference: $(a + bi) - (c + di) = (a - c) + (b - d)i$

*Properties that apply to complex numbers:

Associative Properties of Addition and Multiplication

Commutative Properties of Addition and Multiplication

Distributive Property of Multiplication over Addition

**Complex Conjugates**
**a + bi and a - bi**

Notice: $(a+bi)(a-bi) = a^2 - abi + abi - b^2i^2$
$= a^2 - b^2(-1)$
$= a^2 + b^2$

**Examples:**

1) Find the real numbers a and b such that equation is true.

a + bi = 12 + 5i $\boxed{a=12}$ $\boxed{b=5}$

2) Write the complex number in standard form.

$2i^2 - 6i$

$2(-1) - 6i$

$\boxed{-2-6i}$

3) Perform the operation and write result in standard form.

(a) $\left(\frac{3}{4}+\frac{7}{5}i\right)-\left(\frac{5}{6}-\frac{1}{6}i\right) = \boxed{-\frac{1}{12}+\frac{47}{30}i}$

$= \frac{3}{4} - \frac{5}{6} + \frac{7}{5}i + \frac{1}{6}i$

$= \frac{9}{12} - \frac{10}{12} + \frac{42}{30}i + \frac{5}{30}i$

(b) $-6(5-3i) = \boxed{-30+18i}$

(c) $(6-2i)(2-3i)$ FOIL

$12 - 18i - 4i + 6i^2$

$12 - 22i + 6(-1)$

$\boxed{6-22i}$

(d) $(5-4i)^2 = (a-b)^2 = a^2 - 2ab + b^2$

$= 5^2 - 2(5)(4i) + 16i^2$

$= 25 - 16 - 40i = \boxed{9-40i}$

4) Write the complex conjugate - then multiply the numbers.

$-4-\sqrt{3}i$ $\boxed{-4+\sqrt{3}i}$

$(-4-\sqrt{3}i)(-4+\sqrt{3}i) = a^2+b^2$

$(-4)^2 + (-\sqrt{3}i)^2 = 16 + 3 = \boxed{19}$

HW 2.4a:
page 133
#7-49 odd

# Pre-Calc 2.4b Complex Numbers

## Writing quotients in standard form

To write the quotient a + bi and c + di in standard form, where c and d are not both zero, multiply and numerator and denominator by the complex conjugate of the denominator.

Example 1: $\dfrac{2+3i}{4-2i}$

Example 2: Perform the operation (Hint need LCD)

$$\frac{2i}{2+i}+\frac{5}{2-i}$$

## Recall how to simplify a radical:

$$\sqrt{-24}=\sqrt{-1\cdot 4\cdot 6}=i(2)(\sqrt{6})=2\sqrt{6}i \quad \text{or} \quad 2i\sqrt{6}$$

Example 3: Perform the operation and write in standard form.

(a) $\sqrt{-3}\sqrt{-12}$

(b) $\left(-1+\sqrt{-3}\right)^2$

Pages 133-134

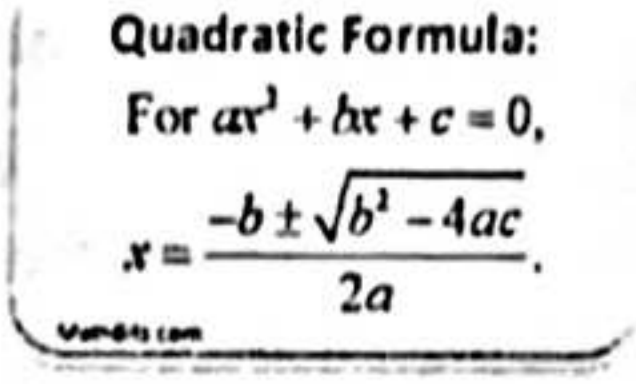
#81-83 odd

# Complex Solutions of Quadratic Equations

Recall that quadratics can have "imaginary solutions"

To solve - use the quadratic formula

$$x^2+4x+5=0$$


**Quadratic Formula:**

For $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

MathsIsFun.com

no x-intercepts

$$x=\frac{-4\pm\sqrt{(4)^2-4(1)(5)}}{2(1)}$$

no "real" solutions $x=\frac{-4\pm\sqrt{16-20}}{2}=\frac{-4\pm\sqrt{-4}}{2}=\frac{-4\pm2i}{2}$

$$\frac{-4}{2}\pm\frac{2i}{2}=\boxed{-2\pm i}$$

Example 4: Solve the quadratic equations below.

(a) $x^2+6x+10=0$

(b) $\frac{7}{8}x^2-\frac{3}{4}x+\frac{5}{16}=0$

HW 2.4b

Pages 133-134

#51-83 odd

Key

# Pre-Calc 2.4b Complex Numbers

## Writing quotients in standard form

**To write the quotient a + bi and c + di in standard form, where c and d are not both zero, multiply and numerator and denominator by the complex conjugate of the denominator.**

Example 1: $\frac{2+3i}{4-2i}$

$= \left(\frac{2+3i}{4-2i}\right)\cdot\left(\frac{4+2i}{4+2i}\right)$ "FOIL"

$= \frac{8+4i+12i+6i^2}{16-4i^2} = \frac{2+16i}{20}$

$= \frac{2}{20}+\frac{16}{20}i = \boxed{\frac{1}{10}+\frac{4}{5}i}$

Example 2: Perform the operation (Hint need LCD) $\longrightarrow (2+i)(2-i)$

$\frac{2i}{2+i}+\frac{5}{2-i} = \frac{2i(2-i)}{(2+i)(2-i)} + \frac{5(2+i)}{(2+i)(2-i)}$

$= \frac{4i-2i^2+10+5i}{(2+i)(2-i)}$

$= \frac{12+9i}{4-i^2}$

$= \frac{12+9i}{5}$

$= \boxed{\frac{12}{5}+\frac{9}{5}i}$

## Recall how to simplify a radical:

$\sqrt{-24}=\sqrt{-1\cdot 4\cdot 6}=i(2)(\sqrt{6})=2\sqrt{6}i$ or $2i\sqrt{6}$

Example 3: Perform the operation and write in standard form.

(a) $\sqrt{-3}\sqrt{-12} = \sqrt{3}i\cdot\sqrt{12}i = \sqrt{36}\,i^2 = 6i^2 = \boxed{-6}$

* Have to take out i before multiply!

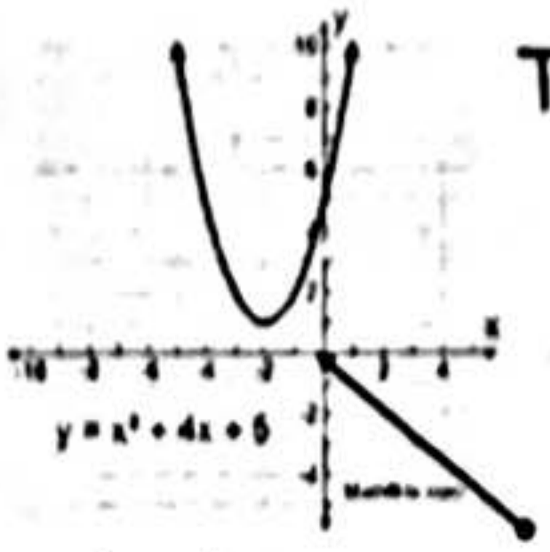
(b) $\left(-1+\sqrt{-3}\right)^2 = (-1)^2 - 2\sqrt{3}i + (\sqrt{3})^2(i^2)$

$= 1 - 2\sqrt{3}i + 3(-1)$

$= \boxed{-2-2\sqrt{3}i}$

# Complex Solutions of Quadratic Equations

Recall that quadratics can have "imaginary solutions"



no x-intercepts

no "real" solutions

To solve - use the quadratic formula

$$x^2 + 4x + 5 = 0$$

**Quadratic Formula:**
For $ax^2 + bx + c = 0$,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2}$$

$$\frac{-4}{2} \pm \frac{2i}{2} = \boxed{-2 \pm i}$$

Example 4: Solve the quadratic equations below.

(a) $x^2 + 6x + 10 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36-40}}{2} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2i}{2}$$

$$= \boxed{-3 \pm i}$$

HW 2.4b

Pages 133-134

#51-83 odd

(b) $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$

Multiply both sides by 16!

$$14x^2 - 12x + 5 = 0$$

$$x = \frac{12 \pm \sqrt{(12)^2 - 4(14)(5)}}{2(14)}$$

$$x = \frac{12 \pm \sqrt{-136}}{28} \quad -1 \cdot 4 \cdot 34$$

$$x = \frac{12 \pm 2i\sqrt{34}}{28} = \boxed{\frac{3}{7} \pm \frac{\sqrt{34}}{14}i}$$